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AdS/CFT and Randall-Sundrum model without a brane

Nihat Sadik Deger and Ali Kaya

Center for Theoretical Physics, Texas A&M University
College Station, TX 77843, USA
E-mail: sadik@rainbow.physics.tamu.edu, ali@rainbow.physics.tamu.edu

Abstract: We reformulate the Randall-Sundrum (RS) model on the compactified AdS by adding a term proportional to the area of the boundary to the usual gravity action with a negative cosmological constant and show that gravity can still be localized on the boundary without introducing singular brane sources. The boundary conditions now follow from the field equations, which are obtained by letting the induced metric vary on the boundary. This approach gives similar modes that are obtained in [1] and clarifies the complementarity of the RS and the AdS/CFT pictures. Normalizability of these modes is checked by an inner-product in the space of linearized perturbations. The same conclusions hold for a massless scalar field in the bulk.

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Any realistic theory of gravity should be able to produce r^{-1} behavior of the gravitational potential. Generically, the potential falls off like $r^{-d_{\rm inf}+3}$, where $d_{\rm inf}$ is the number of dimensions with infinite extend. Thus, to get r^{-1} behavior, higher dimensional theories of gravity have been assuming compactness of the extra dimensions. However, last year Randall and Sundrum [1] showed that this result can still be obtained allowing a non-compact fifth dimension. Their now well-known model (RS) consists of a positive tension 3-brane in AdS_5 which corresponds to our universe and it was shown in [1] that gravity is localized on it with the usual newtonian behavior.

It is reasonable to try to apply the AdS/CFT duality [2]–[4] to understand this phenomena. This has already been pointed out by Maldacena and Witten in their unpublished remarks and has been pursued in several papers [5]–[11]. In these, the brane is thought to be located at finite radial distance which can be considered as the boundary of the AdS space. Geometrically, this gives rise to a compact slice of the AdS space. AdS/CFT correspondence now implies that RS model is equivalent to a four dimensional gravity coupled to a strongly interacting CFT.[1]

However, there is an apparent difficulty in the above considerations. On the RS side, the presence of a singular brane in the *bulk* is very crucial [1, 7]. Indeed, without the brane one has two massless modes and two towers of continuum massive modes on AdS. Introducing the brane, the field equations pick up a delta function source and RS background now becomes a solution.[2] However, once the brane is assumed to be on the boundary, then this delta function is lost since the brane action does not anymore modify field equations which are derived by keeping the metric fixed on the boundary. Therefore, in this case it is not obvious how to repeat the calculations of [1, 7]. On the other hand, on the AdS/CFT side, the role played by this dynamical brane is not clear. For instance, it is not known how to include the degrees of freedom associated with it in the path integral.

In this letter we will show how these problems can be solved by replacing the brane action in the bulk with a term proportional to the area of the boundary of the compactified AdS space. It turns out, in deriving field equations if one assumes that the metric on the boundary is not fixed, then this gives a boundary condition.[3] Indeed, this is not an assumption but a necessity when in the path integral quantization one includes the degrees of freedom associated with the boundary metric. After obtaining the boundary condition, one finds a single massless mode and a tower of continuum massive Kaluza-Klein (KK) modes on AdS. As it will be shown, the massless mode which extends from horizon to the boundary at infinity is not normalizable. However, removing the asymptotic region gives rise to normalizable

[1]For applications of this equivalence to cosmology see, for instance, [12]–[15].

[2]Several no-go theorems have been proved [16]–[20] stating that there are no *smooth* RS compactifications for a large class of supergravity theories.

[3]See also [21] for a different variational approach.

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modes. Thus the RS scenario can be reformulated on a compactified AdS space, which may arise as part of the string compactifications on orientifold and/or orbifold spaces, as discussed in [10]. With this understanding, it is not necessary to assume the existence of a singular, dynamical brane (for instance located at the boundary).

Assuming that such a model arises in a string/M theory compactification, it is natural to consider presence of different fields in the bulk other than gravity. However, in the brane world scenario it is not very clear how to treat, for instance, a scalar field since there is no unique and natural coupling of a scalar to the brane. Another advantage of the approach presented in this letter is that, other fields propagating in the bulk can be treated exactly like gravity; one starts from the well-known action which is used in AdS/CFT, and obtains the equations of motion by allowing field variations on the boundary. To demonstrate this we will consider a minimally coupled scalar field and find, on the boundary, a single massless mode and a tower of heavier, continuum KK modes. Including gravity, AdS/CFT implies a dual description which corresponds to gravity and a scalar field coupled to a strongly interacting CFT on the boundary. In one picture, the graviton and scalar field propagators pick up corrections due to the exchange of KK modes and in the dual picture these corrections arise from the coupling of fields to CFT. Specifically, two-point function of the energy-momentum tensor modifies graviton propagator. On the other hand, the two-point function of the operator in the CFT, which is dual to the scalar at hand in AdS/CFT, gives rise to a correction for the scalar propagator. We clarify the passage from the RS picture to AdS/CFT description by evaluating the same partition function using either AdS/CFT or a semiclassical approximation.

In $d+1$-dimensions, field equations for gravity with a negative cosmological constant Λ are

$$R_{AB}-\frac{1}{2}G_{AB}R=\Lambda G_{AB}\,, \tag{1}$$

which admit the AdS space

$$ds^2=\frac{l^2}{z^2}(dz^2+\eta_{\mu\nu}dx^\mu dx^\nu) \tag{2}$$

as a solution with $l^2=-d(d-1)/(2\Lambda)$. Identifying z as the radial coordinate, AdS space can be viewed to be the metric of a domain wall spanned by the coordinates x^μ. The domain wall will serve as a model for our observed universe and at this moment can be thought to be located at an arbitrary radial position. Let us now discuss if such a scenario can produce the well known properties of gravity without a cosmological constant. We first note that in the absence of matter, the metric on the domain wall is flat. Assuming existence of matter on the wall, one should replace the flat metric $\eta_{\mu\nu}$ in (2) with a curved one $g_{\mu\nu}(x)$. Then, in vacuum, (1) implies the Ricci flatness of $g_{\mu\nu}$. Thus, it seems that one can easily recover the two basic properties of gravity without a cosmological constant.

Let us now consider a similar calculation from an effective action point of view. In $d+1$-dimensions the action is,

$$S = \frac{1}{16\pi G_{d+1}} \int_M \sqrt{-G}\ (R - 2\Lambda)\ + \frac{1}{8\pi G_{d+1}} \int_{\partial M} \sqrt{-\gamma}\ K\,, \tag{3}$$

where G_{d+1} is the Newton's constant and $\gamma_{\mu\nu}$ is the induced metric on the boundary. Since, in deriving field equations (1) the metric is held fixed on the boundary, there is a certain freedom of adding boundary terms to the action. The most natural modification is to add

$$S_1 = \frac{a}{16\pi G_{d+1}} \int_{\partial M} \sqrt{-\gamma}\,, \tag{4}$$

where a is a constant. This term is proportional to the area of the boundary and its presence was first discussed in [22]. It is worth to mention that the area term has been used in the AdS/CFT context for some time. Following [22], the same term was present in the calculation of Weyl anomaly in the paper [24]. This term was motivated further by hamiltonian formulation of the supergravity action in [25]. And finally, it was found by using the counterterm technique in several papers including [26]. Let us emphasize that in these papers this term was *not* considered as a brane source as in our approach in this letter.

For the metric, we will assume

$$ds^2 = \frac{l^2}{z^2}(dz^2 + g_{\mu\nu}(x)dx^\mu dx^\nu)\,, \tag{5}$$

and try to obtain an effective action for $g_{\mu\nu}(x)$. Although the boundary is located at $z = 0$, we cut the asymptotic region at the surface $z = \epsilon$ and name this space as AdS_ϵ. Inserting (5) into the total action $S + S_1$ and choosing

$$al = -2(d-1)\,, \tag{6}$$

we obtain

$$S + S_1 = \frac{1}{16\pi G_d} \int d^dx \sqrt{-g}\ \mathcal{R}\,, \tag{7}$$

where $\mathcal{R}$ is the Ricci tensor of $g_{\mu\nu}$ and the d-dimensional Newton's constant is given by[4]

$$G_d = (d-2)\frac{\epsilon^{d-2}}{l^{d-1}} G_{d+1}\,. \tag{8}$$

Thus, to get a proper effective action describing pure gravity without a cosmological constant, one should start with the total action $S+S_1$ where the free parameter a in S_1 is fixed by (6). Although, this exactly corresponds to the fine tuning in the original RS model, there is an important difference. In the RS model, S_1 has been replaced with

[4] Let us note that the same ϵ dependence in (8) has also been obtained in RG flow considerations, see for example [23].

a singular brane action $S=\tau\int_\Sigma\sqrt{-\gamma}$ coupled to the gravity, where Σ is the world-volume embedded in the bulk. To cancel the contribution of the bulk cosmological constant, the tension τ of the brane should be fine tuned. Thus, the cosmological constant problem is pushed into the brane. However, in the above case, one may expect S_1 with coefficient (6) to come from an AdS compactification of string/M theory. Indeed, let us note that *exactly the same* term should be added to (3) to obtain conformally invariant graviton 2-point function using AdS/CFT duality [22], which shows that in order for AdS/CFT duality to work, such a boundary term should come from an AdS compactification of string/M theory. This raises the possibility of solving the cosmological constant problem not by fine tuning but by the presence of a critical boundary action which may arise after compactification.

From (8), it is clear that ϵ should be kept non-zero. The strength of gravity in d-dimensions is determined by the $d+1$-dimensional Plank scale, cosmological constant Λ and a new length scale ϵ. Choosing ϵ to be very small (and $\epsilon<l$), one can generate a very large Plank mass in d-dimensions from a $d+1$-dimensional Planck mass which may be of the order of standard model energy scales. This is similar to the original proposal of [27] to solve the hierarchy problem and is related to the AdS geometry. Note that this problem cannot be addressed in the second RS model [1]. At this point it is natural to assume the domain wall to be located at $z=\epsilon$. We note that, we do not think of this wall as a dynamical object. It simply means that we live at the boundary at $z=\epsilon$ and by some mechanism we cannot penetrate to the region $z>\epsilon$, at least at low energies.

Although, this scenario seems to work well in d-dimensions, let us mention one potential $d+1$-dimensional problem. From (5), a simple calculation gives [29]

$$R_{ABCD}R^{ABCD}=\frac{2d(d+1)}{l^4}+\frac{z^4}{l^4}\mathcal{R}_{\mu\nu\sigma\rho}\mathcal{R}^{\mu\nu\sigma\rho}\,, \tag{9}$$

where $\mathcal{R}_{\mu\nu\rho\sigma}$ is the curvature tensor of $g_{\mu\nu}$, which indicates a generic curvature singularity on the "horizon" at $z=\infty$. To avoid such a singularity, one may assume the existence of another wall located at a finite z value, which gives rise to a radial coordinate compactified on a line. This is similar to the Horava-Witten construction in 11-dimensions [28], where one coordinate of the flat Minkowski space has been compactified by two 10-branes. In this paper, we do not attempt to modify the discussion by introducing another domain wall. However, it is worth to note that the singular term in (9) vanishes for special backgrounds or may not arise when $g_{\mu\nu}$ in (5) has a non-trivial z dependence. For instance, the gravitational waves propagating along the d-dimensional boundary can be described by plane wave backgrounds, and for such solutions all curvature invariants, such as the singular term in (9), vanish. Also, the strength of gravity due to a static source *on* the boundary turns out to fall down as $z\to\infty$ [7], and there is no singularity problem for this case.

After these effective theory considerations, let us now determine the spectrum of modes on AdS_ϵ. To include the degrees of freedom associated with the gravity on the boundary, one needs to let the induced metric to vary in deriving the field equations. The boundary itself can still be thought fixed, which implies $\delta n_A = 0$ and $\delta n^A = 0$ *on* ∂M, where n_A is the unit normal vector to the boundary.[5] It is not obvious if after these modifications the variation of $S + S_1$ is well defined or if AdS_ϵ is still a solution. To verify this, we vary the action $S + S_1$ by carefully treating the boundary terms coming from integration by parts, and obtain

$$\begin{aligned}\delta(S+S_1) = {} & \frac{1}{16\pi G_{d+1}}\int_M \sqrt{-G}(R_{AB} - \frac{1}{2}G_{AB}R - \Lambda G_{AB})\delta G^{AB} + \\ & + \frac{1}{16\pi G_{d+1}}\int_{\partial M}\sqrt{-\gamma}(K_{AB} - \gamma_{AB}K - \frac{a}{2}\gamma_{AB})\delta\gamma^{AB} = 0\,,\end{aligned} \tag{10}$$

where γ_{AB} is the induced metric on ∂M, K_{AB} is the extrinsic curvature of the boundary defined by $K_{AB} = \gamma_A{}^C\gamma_B{}^D\nabla_C n_D$ and $K = \gamma^{AB}K_{AB}$. In deriving this result, we introduced an adapted coordinate system (gaussian normal coordinates) to the boundary so that the metric near ∂M can be written as

$$ds^2 = \frac{l^2}{z^2}dz^2 + \gamma_{\mu\nu}(x,z)dx^\mu dx^\nu\,. \tag{11}$$

The boundary is located at $z = \epsilon$ and $n = -(z/l)\partial_z$ is the unit normal vector. In this coordinate system, $\delta G_{AB} \to \delta\gamma_{\mu\nu}$ on ∂M (since $\delta n^A = 0$ and $\delta n_A = 0$ imply $\delta G_{AB}n^B = 0$ and $\delta G^{AB}n_B = 0$ on ∂M) and the extrinsic curvature is given by $K_{\mu\nu} = -\epsilon/(2l)\partial_z\gamma_{\mu\nu}|_{z=\epsilon}$. Field equations following from (10) reads

$$R_{AB} - \frac{1}{2}G_{AB}R - \Lambda G_{AB} = 0\,, \tag{12}$$

$$\left(K_{AB} - \gamma_{AB}K - \frac{a}{2}\gamma_{AB}\right)|_{\partial M} = 0\,. \tag{13}$$

Although the bulk equations remain intact, we obtain a boundary condition for the metric. It turns out that AdS_ϵ obeys (13) only if the free parameter a is fixed by (6). Remarkably, the same fine tuning, which has been imposed to cancel the induced cosmological constant on the boundary, is now required to have AdS_ϵ as a solution of the theory.

Assuming small perturbations around AdS_ϵ, the metric of the wall in (5) can be written as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x,z)$. Imposing the gauges $\partial_\mu h^\mu{}_\nu = 0$ and $h^\mu{}_\mu = 0$, the linearized equations following from (12) and (13) read

$$z^2\partial_z^2\hat{h}_{\mu\nu} - (d-1)z\partial_z\hat{h}_{\mu\nu} + m^2z^2\hat{h}_{\mu\nu} = 0\,, \tag{14}$$

$$\partial_z\hat{h}_{\mu\nu}|_{z=\epsilon} = 0\,, \tag{15}$$

[5] This condition can also be imposed as a gauge choice on the boundary.

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where $h_{\mu\nu} = e^{ip.x}\hat{h}_{\mu\nu}(z)$ and m is the d-dimensional mass given by $m^2 = -p^2$. The solutions of (14) obeying the boundary condition (15) are

$$\hat{h} = \begin{cases} \text{const.}; \quad \text{when } \ m = 0\,, \\ \\ N_m(z/l)^{d/2}(A_m J_{d/2}(mz) + B_m Y_{d/2}(mz)); \ \text{ when } \ m \neq 0\,, \end{cases} \tag{16}$$

where N_m is a normalization constant to be fixed in a moment, and $A_m = Y_{d/2-1}(m\epsilon)$, $B_m = -J_{d/2-1}(m\epsilon)$. For $m = 0$, $\hat{h} = z^d$ also solves (14), but does not obey the boundary condition (15). (16) is very similar to the modes found in [1] where, due to the presence of a singular brane, there is an extra delta function source in (14). This implies a boundary condition along the brane eliminating one of the possible zero modes (corresponding to $\hat{h} = z^d$ mode). We see that a similar boundary condition can be obtained from field equations by allowing the induced metric on the boundary to vary, giving again a single massless mode and a tower of continuum massive KK modes in d-dimensions.

At this point, one should make sure of the regularity and the normalizability of these perturbations, which are of the form

$$\delta G_{zz} = 0\,, \qquad \delta G_{z\mu} = 0\,, \qquad \delta G_{\mu\nu} = \frac{l^2}{z^2}h_{\mu\nu}\,, \tag{17}$$

when viewed in $d+1$-dimensional space. Since we remove the asymptotic region, horizon is the only possible location for irregularity. Note that, the Bessel function Y diverges at $z = 0$, therefore the massive KK modes are irregular on AdS boundary when $\epsilon = 0$. The mode $\hat{h} =$ const. represents gravitational waves along the boundary and are regular as discussed below (9). Unfortunately, KK modes seem to be irregular at the horizon. To see this we construct the scalar $\delta G_{AB}\delta G_{CD}G^{AC}G^{BD}$, which diverges like z^{d-1} as $z \to \infty$. This indicates that, contrary to the waves propagating along the boundary, the ones moving through the horizon may be irregular. Let us note that the same type of singularity also arises in the RS model [1]. Whether a divergence of this type has a physical significance is not very clear to us. One may still consider a perturbation of this type to be regular, if the tidal forces on geodesics and all curvature invariants of the perturbed metric turn out to be finite.

It is also interesting to consider the singularity problem in euclidean signature. The massive modes on euclidean AdS_ϵ can be obtained from (16) by analytical continuation $m \to im$, which become a linear combination of the modified Bessel functions $K_{d/2}(mz)$ and $I_{d/2}(mz)$. Although K decays as $z \to \infty$, I diverges exponentially, which shows that the massive modes are singular in euclidean signature. It is possible to overcome this problem by placing another wall at $z = L$. This implies another boundary condition at $z = L$ and m should have discrete values to satisfy this condition.[6] As $L \to \infty$, one recovers the continuum spectrum. Here in this letter, we do not consider the consequences of this modification.

[6] See [30] for a discussion of the scalar field quantization on cut-off AdS at $z = L$.

To check normalizability, one should introduce a suitable inner-product in the space of linearized perturbations. Noting that (14) is the laplacian acting on the scalars, we define

$$\langle h, h' \rangle = -i \int_\Sigma \, (h\, \partial_\mu\, h'^* - h'^*\, \partial_\mu\, h)\, n^\mu d\Sigma\,, \tag{18}$$

where Σ is a $t =$ constant hypersurface, $d\Sigma$ is the induced volume element and n^μ is the future directed unit normal. As we will see, (18) does not depend on the choice of Σ for the modes (16). Using the plane wave dependence of the wave functions along x-directions, one finds

$$\langle h, h' \rangle = (2\pi)^{d-1}\delta(\vec{p} - \vec{p}\,')(\omega + \omega')e^{i(\omega' - \omega)t} I(m, m')\,, \tag{19}$$

where $\omega \equiv p_0$, and

$$I(m, m') = \int_\epsilon^\infty \, dz \frac{l^{d-1}}{z^{d-1}}\, \hat{h}\, \hat{h}'\,. \tag{20}$$

From (20), the massless mode turns out to be normalizable if and only if $\epsilon \neq 0$. Note that, in this case $\vec{p} = \vec{p}\,'$ implies $\omega = \omega'$ and the time dependence in (19) disappears. For massive modes, using the Bessel's differential equation, we obtain

$$I(m, m') = \frac{N_m N_{m'}}{m^2 - m'^2} \left[\frac{z}{l} \left(f_m \frac{d}{dz} f_{m'} - f_{m'} \frac{d}{dz} f_m \right) \right]_\epsilon^\infty\,, \tag{21}$$

where $f_m(z) = (A_m J_{d/2}(mz) + B_m Y_{d/2}(mz))$. There is no contribution from $z = \epsilon$ due to the boundary condition obeyed by $f_m(z)$ at $z = \epsilon$. Evaluating $z = \infty$ limit using the asymptotic forms of the Bessel functions, we find

$$I(m, m') = \frac{\sqrt{2\pi}}{ml}(A_m^2 + B_m^2)\delta(m - m')\,. \tag{22}$$

Therefore, choosing $N_m = (ml/[2\omega(A_m^2 + B_m^2)])^{1/2}$, the massive modes are normalized

$$\langle h, h' \rangle = (2\pi)^{d-1}\sqrt{2\pi}\delta(\vec{p} - \vec{p}\,')\delta(m - m')\,, \tag{23}$$

which is again independent of t. The product of the massless mode with a massive KK mode turns out to be zero since (20) becomes

$$I(m, 0) \sim \int_\epsilon^\infty z\, dz\, f_m\, z^{-d/2} \tag{24}$$

$$\sim \frac{1}{m^2} \left[z \left(f_m \frac{d}{dz} z^{-d/2} - z^{-d/2} \frac{d}{dz} f_m \right) \right]_\epsilon^\infty = 0\,, \tag{25}$$

where we have again used the Bessel's differential equation and the boundary condition obeyed by $f_m(z)$ at $z = \epsilon$. This shows that all inner products are well defined and do not depend on t. Since the set (16) turns out to be *complete* and *orthonormal*, a generic gravitational wave can be obtained by superposing these modes and their complex conjugates. The coefficients of positive and negative frequency modes, $a_m(p)$ and $a_m^\dagger(p)$, are interpreted as creation and annihilation operators on a Fock space of particle states, respectively. Since the background geometry is static, there is no ambiguity in defining the vacuum. The particle created by $a_m^\dagger(p)$ has the mass m and momentum p in d-dimensions. We note that these are valid when $\epsilon \neq 0$. On AdS, when $\epsilon = 0$, the inner products and the Fock space of states are not well defined indicating the decoupling of gravity.

From (23), we see that the measure on the set of continuum eigenvalues is simply dm. Since the dimensionless coordinate on AdS_ϵ is z/l, the corresponding dimensionless eigenvalues and the measure are $l\, m$ and ldm, respectively. Following [1], we can now answer why KK modes below the accessible energy scale of current experiments are not observed. To calculate the probability of creating such a massive mode in a process, we should sum over the continuum eigenvalues up to the energy scale of the process, which is $\int^p (m\epsilon)^{d-3}(\epsilon/l)^{d-1} ldm$, where the factor $(m\epsilon)^{d-3}(\epsilon/l)^{d-1}$ comes from the continuum wave function suppression at $z = \epsilon$. Thus, creation of a massive KK mode is suppressed by a factor of $(\epsilon^2 p/l)^{d-2}$. This is not surprising, since the same conclusion holds for the RS model and the modes found in this paper are similar to the ones found in [1].

The graviton propagator can be calculated either by direct construction or by superposing the modes (16) since they are complete. In the former case, following (14), one needs to solve

$$\Box\Delta(x, z; x', z') = \frac{\delta^d(x - x')\delta(z - z')}{\sqrt{-G}}, \tag{26}$$

$$\partial_z\Delta|_{z=\epsilon} = 0\,, \tag{27}$$

where $\Box$ is the laplacian and the boundary condition (27) is implied by (15). This Green function has been constructed in [7], and is localized near the boundary. Also, assuming the sources to be located on the boundary, Δ can be separated into the standard d-dimensional propagator plus a piece coming from the exchange of KK modes [7], which in momentum space reads

$$\Delta(p) = \frac{d-2}{\epsilon}\,\frac{1}{p^2} \;+\; \Delta_{KK}(p)\,, \tag{28}$$

where

$$\Delta_{KK} = -\frac{1}{p}\,\frac{H^{(1)}_{d/2-2}(p\epsilon)}{H^{(1)}_{d/2-1}(p\epsilon)}\,, \tag{29}$$

and $H^{(1)}$ is the first Hankel function defined by $H^{(1)} = J + iY$.

In the path integral quantization of the system in a semiclassical approximation, one may start from the following integral

$$Z = \int [dG]\, e^{iS \,+\, iS_1}\,, \tag{30}$$

where S and S_1 are given in (3) and (4), and the sum is over *all* metrics on M (no boundary condition imposed on the induced metric on ∂M).[7] This integral can be evaluated using saddle point approximation when there is an extremum of the functional $S + S_1$ under these conditions. As we discussed above, such an extremum should obey (12)–(13), and AdS_ϵ is a solution when a is fixed as in (6). One can then determine the small fluctuations around AdS_ϵ like (16) and construct the propagator of the theory obeying (26) and (27). In this approximation, (30) becomes proportional to the $(\det\Delta)^{-1/2}$.

The path integral approach allows one to see the existence of a complementary picture implied by AdS/CFT correspondence. For this, one can evaluate (30) by first summing over all bulk metrics which match a given boundary metric, and then integrating over all boundary metrics [6]–[8]. This gives

$$Z = \int [d\gamma]\, Z[\gamma]\,, \tag{31}$$

where

$$Z[\gamma] = \; e^{iS_1[\gamma]} \int [dG]_{G|_{\partial M}=\gamma}\, e^{iS}\,, \tag{32}$$

and γ is the induced metric on the boundary. The integral (32) can be calculated by a version of AdS/CFT duality for a finite radial coordinate which, after including necessary counterterms required to have a finite $\epsilon \to 0$ limit, gives

$$Z[\gamma] = e^{iS_1[\gamma]}\, e^{-\frac{i}{8\pi G_{d+1}} \int_{\partial M} \sqrt{-\gamma}\, (\frac{d-1}{l} + \frac{l}{2d-4} R + \cdots)} \langle e^{i\gamma_{\mu\nu} T^{\mu\nu}} \rangle_{CFT}\,, \tag{33}$$

where R is the Ricci scalar of γ and we do not include higher derivative terms. Redefining the metric on the boundary by $g_{\mu\nu} = \frac{\epsilon^2}{l^2} \gamma_{\mu\nu}$, (33) becomes

$$Z = \int [dg] e^{-\frac{i}{16\pi G_d} \int d^d x\, \sqrt{-g}\, \mathcal{R} + \cdots} \langle e^{i g_{\mu\nu} \tilde{T}^{\mu\nu}} \rangle_{CFT}\,, \tag{34}$$

where G_d is given by (8) and the parameter a is again fixed by (6). This shows that, the theory of gravity with a negative cosmological constant defined by the partition function (30) in $d+1$ dimensions is, indeed, equivalent to a d-dimensional gravity without a cosmological constant coupled to a CFT on the boundary. Therefore, complementary pictures arise since the theory defined by (30) can be solved either by a semiclassical approximation giving a set of linearized modes or by first splitting the measure into bulk and boundary parts and then applying AdS/CFT correspondence.

It is straightforward to repeat the above considerations for a minimally coupled massless scalar field on AdS_ϵ. Starting from the action

$$S_s = \int_M \sqrt{-G}\; \nabla_A \phi\, \nabla^A \phi\,, \tag{35}$$

[7] The unit normal vector of ∂M can be fixed by a gauge choice (normal coordinates adapted to boundary). The remaining gauge freedom can be ignored for the following discussion.

and allowing the scalar to vary on the boundary, one obtains

$$\Box\phi = 0$$
$$n^A \nabla_A \phi|_{\partial M} = 0\,. \tag{36}$$

Therefore, in the compactification picture, perturbations around the $\phi = 0$ vacuum obey (14)–(15), and this gives rise to the same modes (16) which consists of a single massless and a tower of continuum massive KK excitations. The propagator obeys (26)–(27) and thus, on the boundary, can be written as (28).

In the AdS/CFT picture, the partition function

$$Z = \int [d\phi]\, e^{iS_s}\,, \tag{37}$$

where the sum is over all fields on AdS_ϵ, can be calculated as

$$Z = \int [d\phi_0]\, [d\phi]_{\phi|_{\partial M}=\phi_0}\, e^{iS_s}\,, \tag{38}$$

where the first sum is over all bulk fields which match a given boundary field, and the second integral is over all boundary fields. Using AdS/CFT this becomes

$$Z = \int [d\phi_0]\, e^{i\int\, d^dx\, \frac{1}{2(d-2)}(\phi_0\, \partial_x^2\, \phi_0 + \cdots)}\, \langle e^{i\phi_0\, O}\rangle_{CFT}\,, \tag{39}$$

where O is the dual CFT operator and the counterterms have been calculated in [31]. The kinetic term in (39) gives the standard $1/p^2$ propagator and the two point function $\langle OO\rangle_{CFT}$ gives rise to corrections.

It is easy to see that when $d = 4$ the first order corrections to the propagators calculated in two pictures agree with each other at large distances. In the case of gravity this has been shown in [6, 7, 9]. In the compactification picture this correction is given by Δ_{KK} in (29) for both gravity and scalar field. On the other hand, in AdS/CFT picture, Ricci scalar in (34) and the kinetic term in (39) give the standard d-dimensional propagator $1/p^2$. From (34) and (39), the first order corrections to this can be calculated to be $1/p^2\langle TT\rangle_{CFT}1/p^2$ and $1/p^2\langle OO\rangle_{CFT}1/p^2$ for the metric and the scalar, respectively. The AdS/CFT correspondence can independently be used to find

$$\langle OO\rangle_{CFT} \sim \frac{1}{x^{2d}} \sim \langle TT\rangle_{CFT}\,, \tag{40}$$

which shows that the first order corrections to the scalar and graviton propagators are also equal to each other in the complementary AdS/CFT picture. Therefore, the agreement found in the graviton propagator in two pictures implies the same conclusion for the scalar field.

In this letter we have showed that it is possible to reformulate alternative to compactification scenario of [1] without introducing singular brane sources. This has been achieved by replacing the brane action in RS model with a term proportional to the area of the boundary of the compactified AdS and letting the metric on the boundary vary. This clarifies the relation between AdS/CFT duality and the RS scenario. The approach presented in this paper also allows one to treat other fields propagating in the bulk exactly like gravity. One important problem which remains to be solved is the singularity of the modes on the horizon of AdS. This problem also arises in the scenario of [1] and may force one to introduce another boundary to cut off $z = \infty$ region. The consequences of such a modification remains to be explored.

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